A Robust Network Structure

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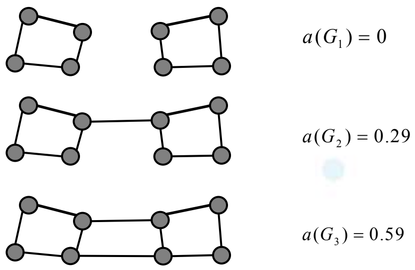
Problem description

Robustness is the ability of a network to avoid malfunction when the network is subject to failures, viruses and other attacks.
Our society depends more than ever on large networks such as the Internet, transportation networks and infrastructures related to energy. We have to understand: How robust a network is?
How to design a robust network?
In this work, we use the algebraic connectivity to quantify network robustness and propose a network structure with given diameter D optimizing the algebraic connectivity.

Robustness quantification by algebraic connectivity

The algebraic connectivity $a(G)$ is the second smallest eigenvalue of the Laplacian matrix of a network G. The Laplacian matrix of G with N nodes is a $N \times N$ matrix $Q = \Delta - A$, where $\Delta = diag(d_i)$ and d_i is the degree of node $i \in N$ and A is the adjacency matrix of G.

The larger the algebraic connectivity is, the more difficult it is to disconnect the network. For example,



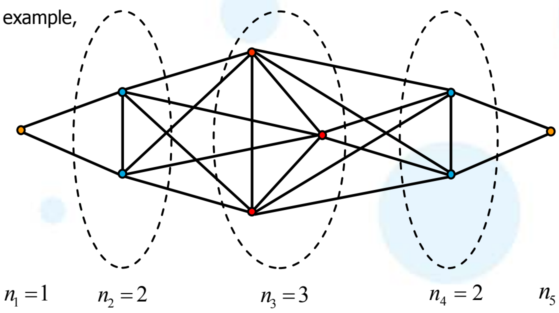
It also characterizes the robustness with respect to synchronization processes and random walks on networks.

Graphs with given diameter D optimizing the algebraic connectivity

Diameter is the maximal distance between nodes in a network. It is crucial for the delay, signal degradation, and spatial spanning of a network.
We design the following network structure, which is actually a class of graphs,

Definition : The class of graphs $G_D^*(n_1 = 1, n_2, ..., n_D, n_{D+1} = 1)$ is composed of $D+1$ cliques $K_{n_1}, K_{n_2}, ..., K_{n_{D+1}}$, where the variable $n_i \geq 1$ with $1 \leq i \leq D+1$ is the size or number of nodes of the i - th clique. Each clique is fully connected with its neighboring cliques $K_{n_{i-1}}$ and $K_{n_{i+1}}$ for $2 \leq i \leq D$. Two graphs G_1 and G_2 are fully connected if each node in G_1 is connected to all the nodes in G_2.

For example,



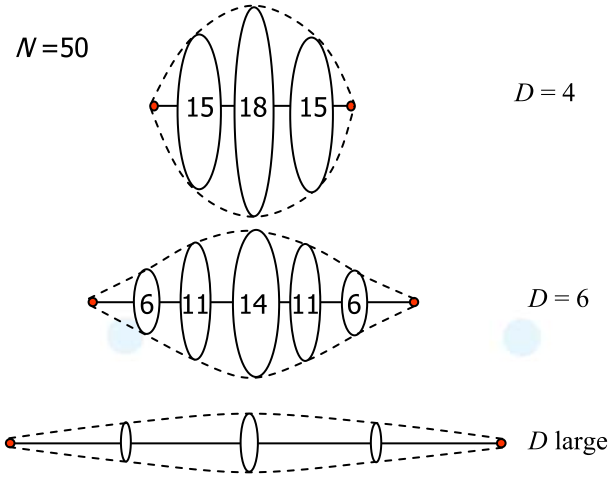
$G_{D=4}^*(n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 2, n_5 = 1)$

A) Properties

Theory : Within the class of graphs $G_D^*(n_1 = 1, n_2, ..., n_D, n_{D+1} = 1)$, the largest number of links, the minimum average distance, and more interestingly, the maximum of any Laplacian eigenvalue among all graphs with N nodes and diameter D can be achieved.

The graph with diameter D optimizing the algebraic connectivity is analytically determined for D<5. For larger diameters, the optimal graph can be found within the class of graphs, because we reduce the Laplacian eigenvalue calculation from a N×N Laplacian matrix to a (D+1) ×(D+1) Jacobian matrix.

B) Example of the optimal graphs



The graph that maximizes the algebraic connectivity is symmetric and has larger sizes for cliques in the middle. It is dense in the core and sparse at borders. Such structures are robust with respect to traffic engineering in the sense that traffic is more uniformly distributed, when traffic is injected between each node pair.

Conclusion

We choose the algebraic connectivity to measure the robustness against failures, because a network possessing a large algebraic connectivity is difficult to disconnect. Furthermore, a large algebraic connectivity may also imply robustness with respect to traffic engineering and dynamic processes like synchronizations and random walks on networks.

Within our structure $G_D^*(n_1 = 1, n_2, ..., n_D, n_{D+1} = 1)$, many robust features can be achieved, e.g. the largest algebraic connectivity, the minimum average distance, etc.

The maximum possible algebraic connectivity can be used to evaluate the upper bounds that have been proposed in mathematics for the algebraic connectivity. Moreover, this robust structure may contribute to the infrastructure design of large networks.

NAS projects on Network Robustness
NWO/Glance: Robunet (643.000.503)
Bsik NGI: Understanding Complex Networks

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