Game Theory of Epidemic Processes on Networks

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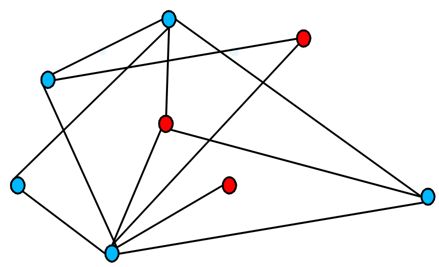
Introduction

The spread of viruses and worms on the Internet is an example of epidemic process where users autonomously decide on protection strategy. An exposed host will become a new source of infection which will attack other unprotected machines on the Internet. Once compromised, host will become a source of information (e-mail accounts, passwords) or direct platform for future attacks. A security breach in one system or host can expose other systems and hosts that had a relation with the compromised system.

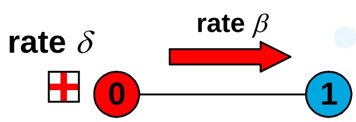


Epidemic spreading – SIS model

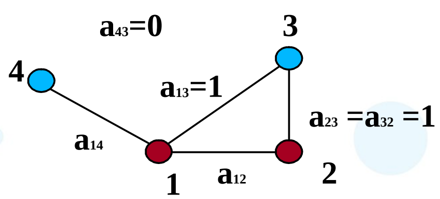
β: virus spreading rate per link

δ: virus cure rate per node

$\tau = \beta/\delta$: effective spreading rate



Adjacency matrix and N-intertwined model



$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}$$

$$\frac{dv_1}{dt} = \beta \sum_{k=1}^{N} a_{1k} v_k (1 - v_1) - \delta v_1$$

$$\frac{dv_2}{dt} = \beta \sum_{k=1}^{N} a_{2k} v_k (1 - v_2) - \delta v_2$$

$$\vdots$$

$$\frac{dv_N}{dt} = \beta \sum_{k=1}^{N} a_{Nk} v_k (1 - v_N) - \delta v_N$$

Game theory

Consider a network with N nodes defined by an adjacency matrix A. This is an underlying topology over which a virus can spread with an infection rate β = 1 per link. Each node i chooses its curing rate, so as to minimize its cost function

$$J_i = c_i \delta_i + v_i$$

where c_i is a positive value that stands for the relative price of protection and quantizes the trade-off of the user between the money (and any overhead) invested in protection and the penalty of being infected.

Two nodes, two protection strategies

- the pure strategy game with two possible strategies

$$\delta_1 = 0.2, \quad \delta_2 = 1.4$$

- Example 1: $c_1 = 1.2, \quad c_2 = 0.9$

Player 1 \ Player 2	δ_1	δ_2
δ_1	(1.04 ; 0.98)	(0.84 ; 1.56)
δ_2	(1.98 ; 0.78)	(1.28 ; 0.86)

Nash equilibr.

- Example 2: $c_1 = 0.3, \quad c_2 = 0.2$

Player 1 \ Player 2	δ_1	δ_2
δ_1	(0.86; 0.84)	(0.66; 1.58)
δ_2	(0.72; 0.64)	(0.02; −0.12)

Nash equilibr.

- Example 3: $c_1 = 0.5, \quad c_2 = 0.6$

Player 1 \ Player 2	δ_1	δ_2
δ_1	(0.9; 0.92)	(0.7; 1.14)
δ_2	(1; 0.72)	(0.3; 0.44)

Nash equilibr.

Related NAS projects

NWO/Glance: Robunet

Bsik NGI: Understanding Complex Networks