Constructing the Overlay Network by Tuning Link Weights

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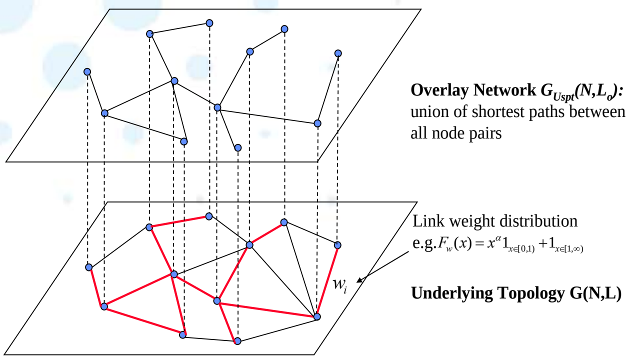
Problem description



The additive and positive link weight measures may represent e.g. the delay, the distance, the monetary cost, etc.

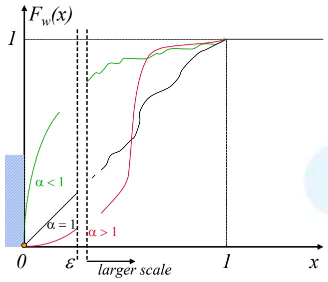
When transport in networks follows the shortest paths or optimized paths, the overlay G_{Uspt} can be regarded as the "transport overlay network", which determines the network's performance. Overlay networks such as peer-to-peer networks or virtual private networks can be considered as a subgraph of G_{Uspt}.

Is the structure of overlay G_{Uspt} controllable via link weights tuning?

Assumptions

• Underlying topology: the Erdös-Rényi random graph $G_{ps}(N)$, where N is the number of nodes and p_s is the link density.

• Each link is equipped with
(a) a single link weight as in best-effort routing: an *i.i.d.* polynomial random variable



(b) a link weight vector as in QoS routing:

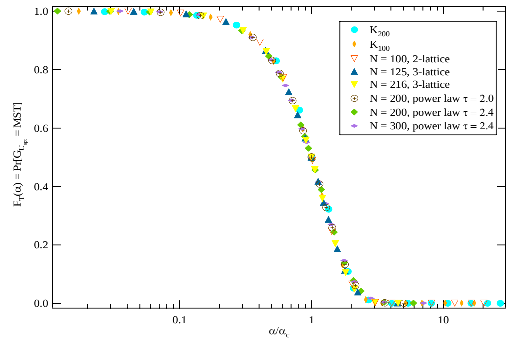
$$\vec{w}(u \to v) = [w_1(u \to v), w_2(u \to v)]$$

where w_1 and w_2 are correlated uniform random variables with correlation coefficient ρ.

Results and Analysis

We construct two kinds of overlay networks: $G_{Uspt(\alpha)}$ by tuning the extreme index α of the one dimensional polynomial link weights and $G_{Uspt(\rho)}$ by tuning the correlation coefficient ρ of the two dimensional correlated uniform link weights.
Structure of the overlay network G_{Uspt} is examined by its number of links, degree distribution and spectrum (eigenvalues of the adjacency matrix).

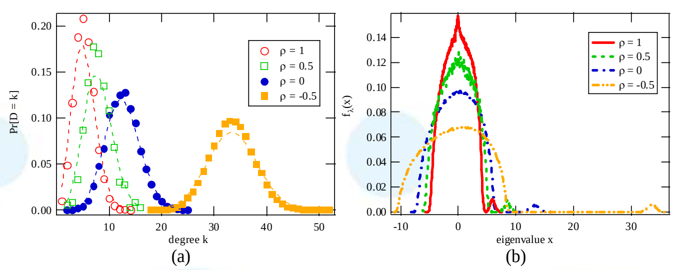
A) One dimensional link weights



The phase transition in G_{Uspt} seems nearly universal for all graphs: the critical α_c scales as $\alpha_c = O(N^{-\beta})$ where $\beta \approx 0.62$. This "link weight" phase transition follows as a power law β which is often observed in nature. If the link weights can be chosen independently of the underlying graph, transport can be switched over two entirely different set of paths. Above α_σ normal transport is obtained which is spread over many links in the network, whereas below α_σ all transport uses the minimum spanning tree (MST).

Conjecture: For large N, the overlay G_{Uspt} on top of a connected Erdös-Rényi random graph $G_p(N)$ with link density $p \in (p_\sigma 1]$ and equipped with *i.i.d.* regular link weights $(\alpha = 1)$ is a connected Erdös-Rényi random graph $G_{pc}(N)$ where p_c is the disconnectivity threshold.

B) Two dimension link weights



Degree distribution and spectrum of the overlay G_{Uspt} on top of K_{100}

The overlay G_{Uspt} is close to an Erdös-Rényi random graph whose link density decreases exponentially as a function of ρ.

Conclusion

one dimension $\alpha \in [0,\infty)$	two dimension $\rho \in [-1,1]$
$G_{Uspt(\alpha<\alpha_c)} = MST$	
$G_{Uspt(\alpha=1)} \approx G_{pc}(N)$	$G_{Uspt(\rho=1)} \approx G_{pc}(N)$
$G_{Uspt(1<\alpha<\infty)} \neq G_p(N)$	$G_{Uspt(-1<\rho<1)} \approx G_p(N)$
$G_{Uspt(\alpha\to\infty)} = G_{ps}(N)$	$G_{Uspt(\rho=-1)} = G_{ps}(N)$

The understanding of the overlay structure with variable link weight structure points to the possibility to control the network structure or to steer or balance transport by tuning the link weight structure.

NAS projects on Network Robustness
NWO/Glance: Robunet (643.000.503)
Bsik NGI: Understanding Complex Networks